Follow-Up

'Three Door' Problem Provokes Letters, Controversy

KENDRICK FRAZIER

It was quite a time! Our Summer 1991 News and Comment article "Nation's Mathematicians Guilty of 'Innumeracy,'" by Gary P. Posner, about Marilyn vos Savant's *Parade* magazine columns on the "three-door problem," has stimulated far more letters to the editor than anything else we have ever published.

The letters began arriving only a few days after the issue reached subscribers, and they didn't begin to let up for many weeks. Even now, months later, new letters come in. Typically an article of special interest might provoke five or six letters; by the time we did the analysis below, we had received more than a hundred letters and the total continues to climb. Few were brief; almost all offered detailed explanations of the readers' views and reasoning. Some offered charts and tables and results from computer programs or tests.

Because we can publish excerpts of only a small fraction of these letters, we took up the offer of puzzle-fan John Geohegan to help us analyze and categorize them. Thirty-seven percent of the readers agreed that vos Savant's solution, "You should switch," was correct; 45 percent (most of whom agreed she was essentially correct, "but . . .") had valid criticisms that her statement of the problem omitted two essential assumptions and that this made a certain answer impossible. Eighteen percent maintained, despite all the explanations given, that she was still wrong; that the odds were equal whether you switched or not. (Several of these readers were quite adamant; some said they thought the article was a hoax. Thankfully, few were as arrogant as the Chicago attorney who wrote: "The contestant has no advantage in switching. . . . Marilyn vos Savant should stick to intelligence *qua* Pop Culture and leave the more serious matters of game-show probabilities to those who paid attention in high school algebra.) Four readers who first maintained emphatically that vos Savant's solution was wrong wrote again later asking that we disregard their previous letters; after further thought or discussions with friends they now understood the problem and realized she was correct. "Please trash that letter," said one. "I have finally figured out what you and she are talking about, and I'd rather not look . . . a fool. . . ." Many readers expressed irritation that vos Savant's statement of the problem didn't make explicit two necessary assumptions (that the host always offers the option to switch and that the host always opens a door with a goat behind it). But almost everyone seemed to enjoy the mental challenge the problem posed. One of my favorite letters gave two pages of discussion and then ended, "I really enjoyed this problem!"

Since our article appeared, several more things have happened. Vos Savant has published a fourth *Parade* magazine column on the subject (July 7, 1991) giving results of her "national test" with letters from schools who responded to her suggestion to test the solution (and revealing indeed a two-thirds chance of winning if you switch). And the *New York Times* published a 2,000-word front-page article ("Behind Monty Hall's Doors: Puzzle, Debate and Answer?" July 21, 1991) that explored the problem and the controversy about it at some length, with comments from game-show host Monty Hall, puzzle expert Martin Gardner, and statistician/probability expert Persi Diaconis. This report, *New York Times* science reporter John Tierney told me while reporting it, was stimulated by the Summer SKEPTICAL INQUIRER article.

Even though we can't publish most of the letters, they will all be useful. Ray Hyman, professor of psychology, University of Oregon, and a longtime member of the *SI* editorial board, has for many years used this kind of problem as a teaching tool in his psychology classes. At his request, we have shared the letters with him. He intends to use them to "tease out the psychological reasons why people have difficulty with this kind of problem."

Here we present John Geohegan's brief comments, followed by excerpts from representative letters (especially those that seem to offer useful explanations), and then a final comment by Marilyn vos Savant. Also note that as a result of all this interest, Martin Gardner has devoted his column in this issue to "Probability Paradoxes."

SI *Readers Show Their Stuff*

JOHN GEOHEGAN

In responding to Gary Posner's comments about innumeracy in the Summer issue, SKEPTICAL INQUIRER readers have demonstrated their competency. We analyzed the first one hundred letters. True, 18 readers disagree with Marilyn vos Savant's answer, but 37 readers agreed, and an additional 45 wrote to explain that the statement of the problem is ambiguous at best. Thus, on a notoriously confusing problem, 82 have presented "right" answers, and 18 "wrong."

One of those who criticized the problem's ambiguity was Robert Meservey of Lexington, Massachusetts: "To obtain Marilyn's result by probability theory, it is necessary to state that the host is *required* to open a second door and give the player a chance to change his choice no matter what the first choice is. Without this stipulation the player might reasonably conclude that the host only offered this choice when the correct door had been chosen already, since a change would then save the sponsor money. In these circumstances a change never wins, whereas no change has 1:3 odds."

Of the 37 who solved the problem as Marilyn intended, one of the most forceful arguments comes from Michael Mauser of Fairbanks, Alaska, who writes: "Suppose the game-show lets you choose any one or any *two* doors, would you choose one door or two doors? This is actually what happens in the original problem statement. You just have to choose the door you believe is *least* likely to have the prize, then switch your choice after given this option."

In *The Second Scientific American Book of Mathematical Puzzles and Diversions*, published in 1961, Martin Gardner describes a slightly different version of this problem as "wonderfully confusing" and "difficult to state unambiguously." True, true.

John Geohegan is chairman of New Mexicans for Science and Reason.

The Problem

Here is the problem as originally published last year in Marilyn vos Savant's *Parade* column. We have inserted in brackets two phrases that might have clarified its implicit assumptions.

> Suppose you're on a game show, and you're given a choice of three doors. Behind one door is a car; behind the others, goats. You pick a door—say, No. 1—and the host, who knows what's behind the doors, opens another door—say, No. 3—which has a goat. [In all games, he opens a door to reveal a goat.] He then says to you, "Do you want to pick door No. 2?" [In all games he always offers an option to switch.] Is it to your advantage to switch your choice?

Readers' Comments

I admit that when I first read Marilyn vos Savant's answer to the game-show question, I was convinced that she was wrong. At least I had some very impressive company. The problem is really quite simple. When you choose your door, you divide the set of doors into two subsets: set C (chosen—containing 1 door) and set U (unchosen—containing 2 doors). Since the likelihood of the car being behind each door is equal, then it is twice as likely that set U contains the door hiding the car. If you were asked at this point whether you wished to stay with your choice of set C or switch to set U, clearly you should switch. By opening a door that he knows does not hide a car, the host only confirms that not all doors hide cars. This was already assumed and has no bearing on whether set C or set U is more likely to contain the door hiding the car. Thanks for a fascinating problem.

Patrick Harren
Houston, Tex.

I was thoroughly entertained by the flap. Vos Savant correctly answered that your chances of winning double if you switch, but why this is true is far from intuitive, as the number of outraged letters attest to. I have come up with what I think is the easiest way to convince the skeptical.

As vos Savant does, let us consider a hundred doors just to make the point with more emphasis. The puzzle then involves the contestant selecting a door, Monty revealing what is behind 98 of the remaining 99 doors, and the contestant being invited to switch. Now let's change the rules ever so slightly. Consider what would happen if, after you select your door, Monty invites you to stick with it, or to take what is behind *all the other doors combined*. Clearly, you would switch, as your first pick gives one chance in 100 of winning whereas taking all other doors now gives you 99 chances in 100. Surprisingly, this game is completely equivalent to the original.

Robert P. J. Day
Calgary, Alberta
Canada

You and Marilyn vos Savant are wrong. The set of facts upon which the question is based do *not require* the game show host to offer the contestant a second choice. The host seems free to offer the choice or not, at his option. Since he has prior knowledge of the winning door, random chance is no longer involved. Calculating the odds is not only irrelevant but impossible.

Paul Kelly
Boulder, Colo.

A colleague raised the "three-door" problem during the "Promoting Critical Thinking" discussion in a workshop I offered on teaching psychology. Most of the audience (and, truth to tell, I myself) were, like many of the mathematicians quoted in the Summer issue, resistant to the counterintuitive logic of the correct answer (which is that switching is more likely to win the car than not switching). I came up with the following account of why the "obvious" answer to the problem is wrong:

No matter which door the contestant picks, the chances of having chosen correctly is 1 out of 3 (33 percent), while the chances that the car is behind one of the unchosen doors is 2 out of 3 (67 percent). Thus the chances of winning the car would immediately improve if the contestant could switch from the lower probability category (the chosen door) to the higher probability category (the unchosen doors). Since there are two doors in the higher probability category, however, it is not clear which of them to pick until the host reveals which of them is a loser. The contestant now knows which one to choose in order to increase the chances from 33 percent to 67 percent.

Douglas A. Bernstein
Professor and Associate Head
Department of Psychology
University of Illinois
Champaign, Ill.

The News and Comment note about supposed innumeracy among mathematicians was quite misleading, as no real mathematical disagreement was involved. Most mathematicians, I think, would now agree that the original statement of the problem did not contain enough information for a solution. The published solution assumed that the host in the game show would always open a losing door, but that was not actually stated in the problem. If you make a different assumption about the underlying situation, you get a different answer.

A computer program incorporating one assumption about the rules would rightly be rejected by those making a different assumption.

William C. Waterhouse
Department of Mathematics
Pennsylvania State University
University Park, Pa.

Wait a minute. Where does the problem say that the host opens door No. 3 because it does not have the prize? Well, it doesn't. For all we know, the host might have opened door No. 3 first simply because he *always* opens door No. 3 first, prize or no prize. The way the problem is written, there is no reason to assume any connection between the door that the host opens and his knowledge of what's behind it.

I found myself troubled by the excerpts I read. Troubled because so many highly intelligent people were reduced to trading snide, sardonic little remarks and wielding computer programs and probability grids, all for a simple little puzzle that just wasn't worded very well. Didn't anybody try to read it both ways?

Albert Klumpp
Arlington Heights, Ill.

There are, I think, four levels of understanding of this brainteaser:

1. No knowledge of the laws of probability; yields simple bewilderment.

2. Knowledge of the laws of probability, but not taking into account that the game-show host gives away information by opening the second door; yields the irate contention that it is 50/50 either way.

3. Knowledge of the laws of probability, and taking into account that the game show host gives away information, but assuming without basis that he is obliged to offer the contestant the option to switch in every case; yields the contention that the contestant should take the offer to switch.

4. Understanding all of the above, realizing that the game-show host has every reason to refrain from offering the contestant the option to switch except when the contestant has chosen the car on the first try; yields the contention that the contestant should refuse the offer to switch.

Tim Stryker
Fort Lauderdale, Fla.

I am afraid the only ones guilty of "innumeracy" are you, your magazine, vos Savant, Posner, reader Lenna Mahoney, and the rest of vos Savant's supporters on this issue, not the "nation's mathematicians." Posner should stick to medicine and leave mathematics to those trained in it. Mahoney had better check her simulations and make sure that her atmospheric science background didn't "cloud" her programming logic.

The *switch* strategy does not lead to any advantage whatsoever despite vos Savant's mumbo-jumbo of an explanation.

W. Allen Cochrane
Smyrna, Ga.

I suspect this item was intended for your April Fool edition. . . . In the end there are only two doors, and only one of them leads to the car. At that time, the odds are 1:2 regardless of how that situation evolved.

Suppose you flip a coin 999 times, and it comes up heads every time. On the 1,000th flip, the odds of getting another head are still 1:2. If you want to determine the odds of getting 1,000 consecutive heads, then you may properly consider the preceding 999 flips.

Jimmie R. Osburn
Savannah, Tenn.

The point overlooked by all those critics of Marilyn vos Savant is that the host of the show knows where the car is. By opening a door he knows will render a goat, the host imparts some of his knowledge to you, the contestant. If the host doesn't know which door the car is behind and just happens to open a door with a goat, the decision whether to switch is 50-50.

Donald Keith
Waterloo, Ontario
Canada

It is entirely possible that the various mathematicians who erred on this problem had forgotten that the host was not choosing the door or doors at random. A devilish little problem for the unwary!

Christopher D. Allan
Alsager, Stoke-on-Trent
England

Boy! You guys really had me going. First of all, I did not accept your explanation of why it is to a player's advantage to switch his or her choice after being shown a door that did not have the prize behind it. I was convinced the odds were 1/2.

With calm assurance, I wrote a short computer program to simulate the problem. I was certain that in 10,000 trials, I would see about 5,000 hits if the player switched and 5,000 if he or she didn't. When I ran the program, the results: 6,679 wins on switching, 3,321 on standing pat! I was stunned. Had I made a programming error, however unlikely that might be? No, the code was fine. So why was there a 2/3 likelihood of success upon switching? Upon further analysis, I determined that the code and the 2/3 probability were both correct. However, my explanation is different from the ones that appeared in the SKEPTICAL INQUIRER (which I still do not find compelling).

Rather than being a problem in probability (a red herring), it is a problem in Boolean logic. Succinctly put, a switching strategy on the second guess negates the outcome of the first guess. Since the odds of making a *wrong* choice on the first guess are 2/3, you have a 2/3 probability of winning with this strategy.

Joseph G. Dlhopolsky
Port Jefferson Station, N.Y.

There are a number of lessons to be learned from the confusion generated by the "brainteaser." One is that apparently simple problems in probability can be confusing, even to well-trained mathematicians. Anyone familiar with the history of probability theory can produce examples of eminent mathematicians, especially prior to 1660, who produced wrong answers to "elementary" problems in probability theory. A second lesson is that it is usually a good idea when defending one's own solution to these kinds of problems to avoid high levels of self-confidence. A third and final lesson is that, as with any discipline, it is wise to consult the relevant literature before attempting to reinvent the wheel. Steve Selvin provides a brief discussion of the brain-

Marilyn vos Savant Comments

I'm glad you're enjoying the game-show controversy.

Your readers may be interested to know that virtually all of my critics understood the intended scenario! I personally read nearly 3,000 letters [to *Parade*] (out of the many additional thousands that arrived) and found nearly every one insisting simply that because two options remained (or an equivalent error), the chances were even. Very few raised questions about ambiguity, and the letters actually published in the column were not among them.

But for those readers now interested more in the analysis of ambiguity, let me offer the following notes. When I read the original question as it was sent by my reader, I felt it didn't emphasize enough that the host always opens a door with a goat behind it, so I added that to the answer to make sure everyone understood. And as for whether the host offers the switch each time, I don't see that as a valid objection. It wasn't offered as a factor, so the original is the paradigm. The contestant chooses a door each time; the host opens a door each time. (The contestant doesn't choose a door and *open* it next time; the host doesn't open the *contestant's* door next time; the host doesn't offer the contestant *money* next time.)

Just because a similar game show appeared on television doesn't warrant the assumption that the published problem involves considerations as subjective as creating audience excitement or saving the sponsor money!

But as the response from your readers shows, it's all been great fun, and we've certainly learned a lot.

Marilyn vos Savant
Parade Publications, Inc.
New York, N.Y.

teaser (also known as the Monty Hall problem) in the Letters to the Editor section of the *American Statistician*, February and August 1975.

Robert S. Lockhart
Toronto, Ontario
Canada

I do not understand why it should take eminent mathematicians from MIT, and computer programs, to verify Marilyn vos Savant's answer. A simple explanation, like the following one, should be sufficient to clarify the matter:

You have three doors A, B, and C. Assume the car is behind B. If you pick A, then C must be opened, and if you switch to B, you win. If you pick C, then A must be opened, and if you switch to B you win. If you pick B, then either A or C must be opened, and if you switch to either one, you lose.

Thus, no matter behind what door the car is, if you switch your chance to win is always 2/3.

However, as vos Savant correctly deduces, once the switching possibility is removed, the chance to win the car is 1:2.

Bernhard H. Schopper
Alexandria, Va.

If I were playing vos Savant's game I would switch every time. That way I would lose each time I chose the car initially (one time in three tries). I would win each time I chose a goat initially (twice in three tries).

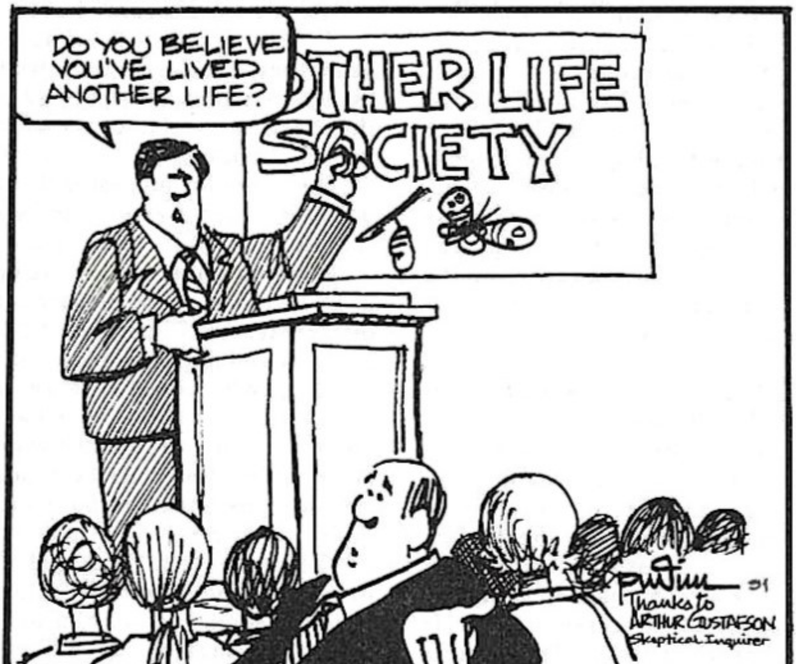
And I wouldn't give a damn for mathematics or probabilities.

Blake Matthews
Winston-Salem, N.C.

I was amused by the elaborate attempts to explain the best strategy to use in the game-show contest popularized in Marilyn vos Savant's *Parade* column and discussed in your article. ("It wasn't until I started writing a computer program. . . .") If I am the contestant and my strategy is always to stick with my initial choice, then the probability that I *win* is 1/3. If my strategy is always to switch, then the probability that I *lose* is 1/3, since I lose only if the car is behind the door I first choose. As I either win or lose every time I play, the probability that I win using the switching strategy is 2/3.

James G. Simmonds
Department of Applied
Mathematics
University of Virginia
Charlottesville, Va.

OUT THERE Rob Pudim



HUGO HOGMIRE DECIDES NO, BECAUSE OF ALL THE STUPID MISTAKES AND BIG-TIME SCREW-UPS HE'S MADE IN THIS ONE.